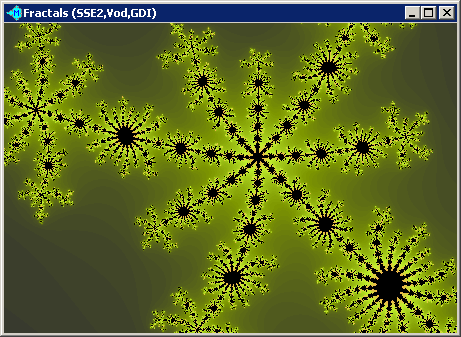
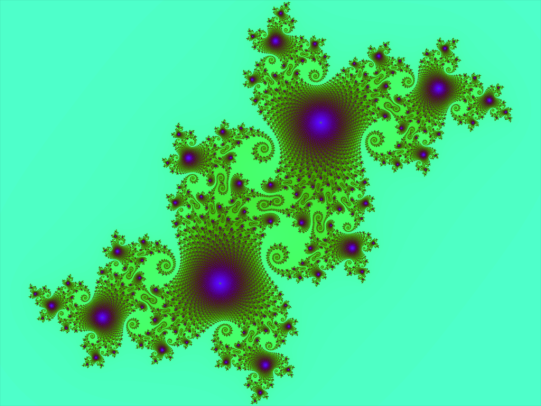
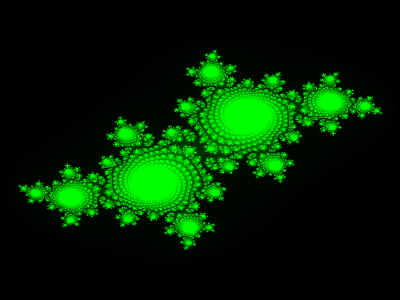
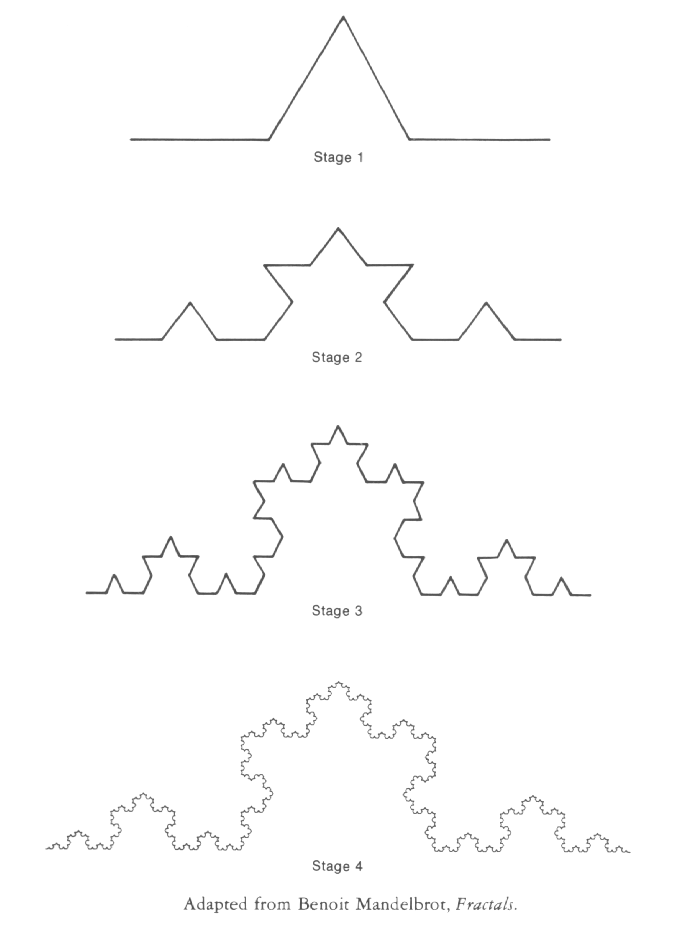
**FRACTALS**

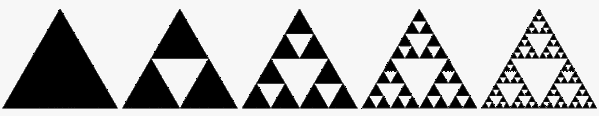
1. Take a look at these Fractals displayed below. Describe what you see in each individual picture. Are there similarities? Differences?
2. In art there is a great deal of attention on detail, creating pictures that are very intricate. In mathematics pictures can be produced to literally infinitely intricate. Images can be drawn to any point of detail that our finest printers can come up with. Not everything can be seen by the naked eye though; with every magnification another great detail is revealed. Other images reveal unthought-of variations and surprises at ever-increasing levels of magnification. Images in the mind are created by following a pattern that is repeated over and over. One example would be when you get your hair cut and they use the two mirror method to show you the back of your head. When you look into the mirror it looks like you can see repeated images going on forever. Lets explore some everyday life examples.

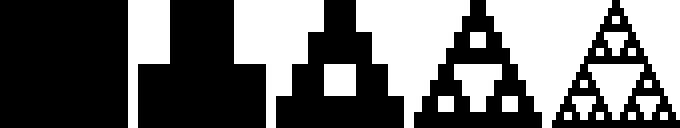
Lets look at the mirror example that we mentioned before. Hold two mirrors, one a hand mirror and one a regular wall mirror. Hold the hand mirror up in the regular one so you can see an image. What do you see? - You should be able to see a repeating process, if not position yourself so you are. Look into the small mirror; you should be seeing what is called a repeated process or repeated image. That image would have infinite detail, because each image of the hand mirror would contain in it an image of a yet smaller hand mirror and it can continue on forever. Mirrors are great ways to show a concrete way to think about images with infinite detail.

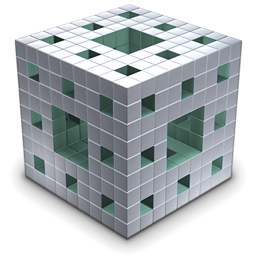
Another example of Repeated Placement is with what is called the Koch Curve. This can begin with a line segment that has four equal lengths, each one a third as long as the original. This will show as a line segment with a kink in the middle. Now we are going to do it again. Replace each segment with four segments each a third of the length of the previous segments, like before. We can continue doing this over and over and over until we end up with wiggly lines that to the naked eye don’t have any meaning, but if you place one under magnification what will happen? You can see that it expresses exact self-similarity. Looking at a part of the Koch Curve you cannot tell if you are seeing it in actual size or magnified a million times. It looks exactly the same.

🡨Example of a Koch Curve

Sierpinski Triangle

Waclaw Sierpinski first described his fractal triangle in 1916. Begin with a filled in triangle with a similar upside down triangle in the middle of it. This gives you 3 filled in triangles with one in the middle that is not filled. For each filled in triangle place an upside down empty triangle in the middle of it and repeat. The resulting object is called the Sierpinski Triangle.

The image guarantees this self-similarity because the process of replacement is identical for each sub-triangle as it was for the whole triangle. Now lets begin with a filled –in equilateral triangle and then make three smaller copies, each half as tall and half as wide as the original. They should be placed in a row. These three reduced copies and their positions specify this collage- generating instruction set. Take the result picture; take three reduced copies, place them in a line again. Continue over and over. You can now see that the Spierinski Triangle has infinitely many holes. We were exactly reconstructing the Spierinski Triangle. After mastering these steps lets try a different shape and see what we get. Begin with the first set of constructions but begin with a square this time. The first few steps don’t look like we are going in the right direction but keep continuing the pattern. After several repeats of the process our image is becoming closer to looking like a Sierpinski Triangle. If you look closely you can see the small squares creating the final image.

Lets try a three dimensional example of repeated placement. One example can be a Menger Sponge. A solid cube can be replaced by a 20 solid sub-cube. Continue by replacing each of those 20 cubes by 20 sub-cubes yields 400 yet smaller cubes. This can be continued over and over resulting in a Menger Sponge.

**The Chaos Game!**

Lets try a game to further our understanding of fractals. Begin by numbering the vertices of an equilateral triangle 1, 2, and 3. Start at a random vertex. You can use a die for this part and let 1 or 4 mean 1, 2 or 5 mean 2 and 3 or 6 mean 3. Roll the dice and move halfway from where you are toward that numbered vertex and make a dot an remain there. Repeat the process again and again. This creates a sequence of dots. As you begin to make the dots you will notice that the dots do not become a random collection, but instead they begin to form a pattern. If we kept continuing this for many steps, we will see the Sierpinski Triangle appear.

1. Suppose we take a point in the plane and draw the line segment from that point to the origin. Then measure the length of the segment and angle going counter clockwise starting from the positive x-axis to the segment. We will not find a new point. The new point we get will have distance from the origin equal to the square of the length of the segment. The square of the distance of the original point to the origin-and will be in the direction at twice the angle of the original segment. To put it simply, you double the angle and square the length.

This brings us to Squaring Practice and a Julia Set warm up. Those pictures result from an interactive process where all we are doing is repeating, repeating, and repeating.

To get started lets practice some arithmetic of complex numbers. Begin with 2+3i, square it, take that new number and square it, take that result, square it and keep doing that. What do we get?

(2+3i)(2+3i)=-5+12i

With our repeated process we come up with…

(-5+12i)(-5+12i)= (25-144) + [2(-60)]i=-119-120i

If we keep repeating this process we come up with…

-5+12i

-119-120i

-239+28560i

-815616479-13651680i

And it continues with larger numbers.

So when you are plotting these on a graph you are going to need to make it small because the points that you have are moving farther away from the origin.

1. Edward N. Lorenz a meteorologist one day saw the outcome of what happens when what you think is a slight change in numbers. It was a discovery by accident one day. In the 1960’s his computer crashed after doing research. He had to restart the computer at an intermediate point. To save time he decided that he was just going to round the parameter values rather than copy the decimals number for number. He noticed that the answers he got were radically different from those he had gotten when he ran the actual numbers. He investigated that the weather predictions came out entirely different when he rounded the parameters to two decimal places rather than using the three or four. Just a tiny change can make a huge difference in your outcome.